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WHY MULTIVARIATE ANALYSIS?

- Used for analysing complicated data sets
- When there are many Independent Variables (IVs) and/or many Dependent Variables (DVs)
- When IVs and DVs are correlated with one another to varying degrees
- When need to come up with Prediction Model
- Parallels greater complexity of contemporary research

USING MULTIVARIATE ANALYSIS

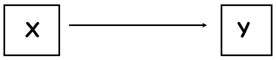
- WHICH STATISTICAL PROCEDURE TO USE?
- HOW TO PERFORM CHOSEN PROCEDURE?
- HOW TO INFER FROM RESULTS OBTAINED?
- ANY OTHER ALTERNATIVE APPROACH?

CHOICE OF APPROPRIATE STATISTICAL METHOD BASED ON:

- Nature of IVs and DVs
- Investigator's Experience
- Personal Preferences
- Ease of Comfort with Methods Used
- Literature Review References
- Consultation with Statistician

BASIC PREMISE I:

Relationship between



Eg . Smoking → Lung Cancer
Age → Hypertension
Maternal ANC → Birthweight

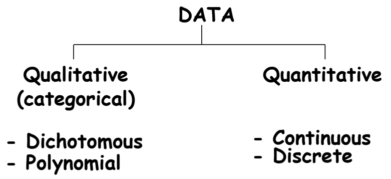
INDEPENDENT VARIABLE	DEPENDENT VARIABLE
RISK FACTOR	OUTCOME

BASIC PREMISE I:

TERM USED:-

X	Y
Independent Variable (IV)	Dependent Variable (DV)
Predictor	Outcome
Explanatory	Response
Risk Factor	Effect
Covariates (Continuous)	
Factor (Categorical)	
Control	
Confounders	
Nuisance	

BASIC PREMISE II:



TERMINOLOGY:

Univariate Analysis

- Analysis in which there is a single DV

Bivariate Analysis

- Analysis of two variables
- Wish to simply study the relationship between the variables

Multivariate Analysis

- Simultaneously analyse multiple DVs and IVs

BASIC PREMISE 111:VARIATIONS OF THE SAME THEME

THE GENERAL LINEAR MODEL:

$Y = a + b_1x_1 + b_2x_2 + b_3x_3 \ldots. + b_ix_i$

Used in the following procedures:

ANOVA

ANCOVA

Multiple Linear Regression

Multiple Logistic Regression

Log Linear Regression

Discriminant Function

Rough Guide to Multivariate Methods (1)

Name	Xs	Y
Regression and Correlation	Continuous (eg. age)	Continuous (eg. BP)
Analysis of Variance (ANOVA)	Categorical (eg. SES)	Continuous (eg. BP)
Analysis of Covariance (ANCOVA)	Categorical and Continuous (eg age and SES)	Continuous (eg BP)

Rough Guide to Multivariate Methods (2)

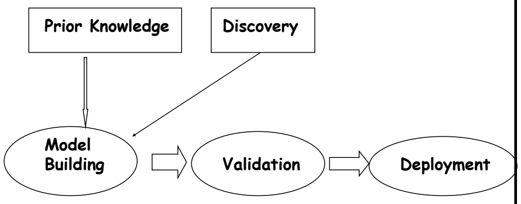
Name	Xs	Y
Multiple Linear Regression	Continuous (eg. Age, Ht, Wt)	Continuous (eg. BP)
Logistic Regression	Continuous (eg. age)	Categorical (eg. CHD)
Logistic Regression	Categorical (eg. sex)	Categorical (eg. CHD)
Discriminant Function Analysis	Continuous (eg. Age, Income)	Nominal / Ordinal (eg. Quality of Life)

BASIC PREMISE IV:

TWO APPROACHES TO CHOOSING:

1. Based on Type of Modelling
2. Based on Type of Research Question

MODELLING:


Prior Knowledge

Discovery

Model Building

Validation

Deployment

TWO TYPES OF MODELLING TOOLS:

1. Theory Driven-Hypothesis Testing:
   Attempts to substantiate or disprove preconceived ideas
2. Data Driven:
   Automatically creates model based on patterns found in data

THEORY-DRIVEN MODELLING TOOLS:

CORRELATIONS

t-TESTS

ANOVA

LINEAR REGRESSION

LOGISTIC REGRESSION

DISCRIMINANT ANALYSIS

FORECASTING METHODS

DATA-DRIVEN MODELLING TOOLS:

CLUSTER ANALYSIS

FACTOR ANALYSIS

DECISION TREES

DATA VISUALISATION

NEURAL NETWORKS

BASIC PREMISE IV:

Types of Research Questions

- Degree of Relationship among Variables
- Significance of Group Differences
- Prediction of Group Membership
- Structure

RESEARCH QUESTION I:

Degree of Relationship among Variables

Statistical technique:

a. Bivariate r (Bivariate correlation and Regression)

b. Multiple R (Multiple Correlation and Multiple Regression)

c. Sequential R

d. Canonical R

e. Multiway Frequency Analysis

RESEARCH QUESTION II:

Significance of Group Differences

Statistical Techniques:

a. t-test
b. One-way ANOVA
c. Two-way ANOVA
d. Profile Analysis

RESEARCH QUESTION III

Prediction of Group Membership

Statistical technique

a. Discriminant Function

b. Multiway Frequency Analysis (Logit)

c. Logistic Regression

RESEARCH QUESTION IV

Structure

Statistical technique:

a. Principal Component Analysis

b. Factor Analysis

c. Structural Equation Modelling

PRELIMINARY CHECK OF DATA BEFORE MULTIVARIATE ANALYSIS

- Accuracy of Data File
- Honest Correlation
- Missing Data
- Outliers
- Normality, linearity and homoscedasticity
- Multicollinearity and Singularity
- Common Data Transformations

Regression Diagnostics

ACCURACY OF DATA FILE :

Inspect univariate descriptive statistics for accuracy of input

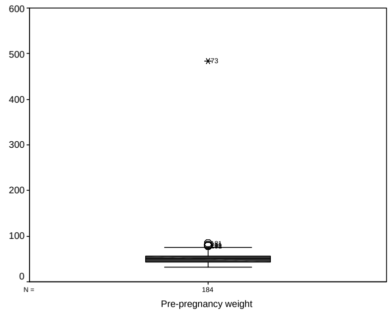
a. Out-of-range values

b. Plausible means and standard deviation

c. Coefficient of variation

d. Univariate outliers

Univariate outliers



HONEST CORRELATIONS

- Inflated Correlation
- Deflated Correlation
- Inaccurately Completed

Inflated Correlation

- If composite variables are to be used and two or more composite variables have the same raw data, correlation can be inflated.
- i.e. correlation between BMI and weight

Deflated Correlation

1) the range of values for one variable is restricted; “relationship between annual average daily traffic count (AADT) and accidents on rural highways and picks a remote region where all AADT's are less than 3000, then if there is a good correlation, he is likely to underestimate it with such a restricted range on one variable.
2) if an intervening variable mediates between two variables; “relationship between thickness of asphalt and chloride content, then picking only those bridges in the population which have a waterproofing membrane will likely push down the estimate. The waterproofing membrane intervenes, literally and statistically. ”

Inaccurately Completed

- Questionnaires inaccurately completed due to lack of time, lack of concern, emotional bias will affect correlation.

◀

MISSING DATA

Seriousness depends on

- Pattern of missing data → Random / Non-Random
- How much is missing
- Why is it missing

MISSING DATA

How to handle missing data

a. Deleting cases or variables

b. Estimating missing data

- use of prior knowledge
- inserting mean values
- using regression

◀

c. Using a missing data correlation matrix

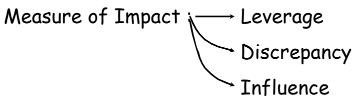
d. Treating missing data as data

e. Repeating analyses with and without missing data

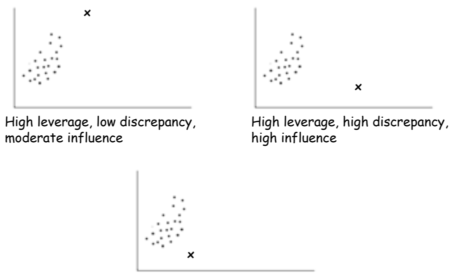
OUTLIERS

- Cases with such extreme values on one variable or a combination of variables that they distort statistics
- Presence due to
  - incorrect data entry
  - failure to specify missing value codes
  - outlier is not a member of target population
  - distribution for variable in population is more skewed than normal

OUTLIERS :


Measure of Impact → Leverage, Discrepancy, Influence

The relationship among leverage, discrepancy and influence


High leverage, low discrepancy, moderate influence

High leverage, high discrepancy, high influence

Low leverage, high discrepancy, moderate influence

OUTLIERS :

Dealing with Outliers

- Find and remedy errors in data entry
- Find and remedy missing values specification
- Deletion
- Retention with alteration

OUTLIERS :

Detecting Outliers

Univariate outliers:

- inspection of z-scores
- graphical methods e.g. histograms, box plots, normal probability plots

Multivariate outliers:

- computation of Mahalanobis distance

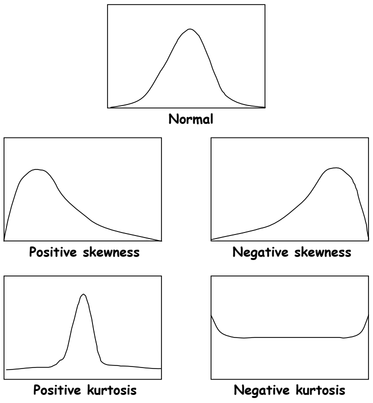
NORMALITY, LINEARITY AND HOMOSCEDASCITY

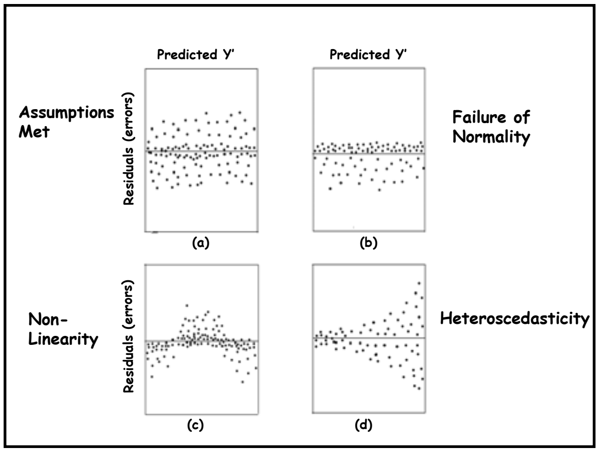
Need for Multivariate Normality

- assumption that each variable and all linear combinations of the variables are normally distributed
- robustness to violation of assumption still inconclusive

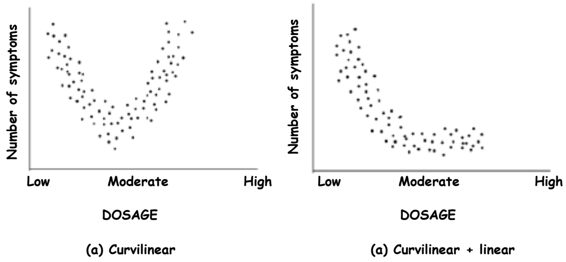
NORMALITY

- assessed via statistical or graphical methods
- 2 components: skewness and kurtosis
- if non-normal, consider transformation





Curvilinear Relationship



LINEARITY

- assumption of a straight line relationship between 2 variables
- Nonlinearity is diagnosed from
  - residuals plots or
  - bivariate scatterplots

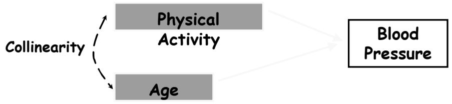
HOMOSCEDASTICITY

- assumption that the variability in scores for one continuous variable is roughly the same at all values of another continuous variable
- failure due to
  - non normality of one of the variables or
  - one variable is related to some transformation of the other

COLLINEARITY

- concerns the relationship of the IVs to one another and does not directly involve the response variable

eg.



PRESENCE OF COLLINEARITY CAUSES

- Unstable regression coefficient estimates
- Large estimates of coefficient variances
- Wide 95% Confidence Limits
- Large p-values
- Large Standard errors

MULTICOLLINEARITY AND SINGULARITY (related to a Correlation Matrix)

Multicollinearity : Variables are too highly correlated (> 0.90)

Singularity : Variables are redundant; Matrix cannot be inversed
the variables are perfectly correlated.

expose the redundancy of variables and the need to remove variables from the analysis.

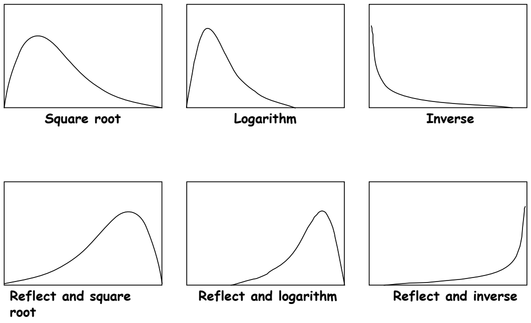
PROBLEMS WITH MULTICOLLINEARITY AND SINGULARITY

- inflate size of error
- weaken analysis

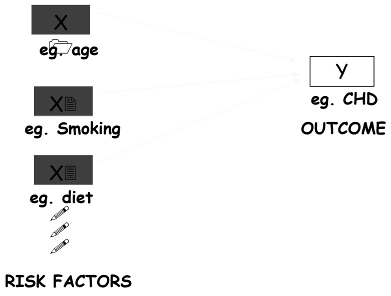
COMMON DATA TRANSFORMATION

• recommended as a remedy for outliers and for failures of normality, linearity and homoscedasticity

TYPES OF TRANSFORMATIONS


Square root
Logarithm
Inverse
Reflect and square root
Reflect and logarithm
Reflect and inverse

MULTIVARIATE RELATIONSHIP



APPLICATIONS OF MULTIVARIATE ANALYSIS

a) The primary purpose is to study the effect on variable Y of changes in a particular single variable X_1, but it is recognised that Y may be affected by several other variables X🗎,X🗏, … The effect on Y of simultaneous changes in X🗁,X🗎, X🗏, … must therefore, be studied.

b) Which of a set of variables X🗁, X🗎, X🗏, … has apparently most influence on the outcome variable Y.

c) To predict the outcome (i.e. variable Y) in future individuals.

MATHEMATICAL MODEL - the General Linear Model

{Outcome} = β🗀 + β🗁X🗁 + β🗎X🗎 + …

⇓

Y in Linear Regression

Logit P in Logistic Regression

ln { h(t) / h🗀(t) } in Cox Regression for survival analysis

β𝓗 measures the relationship between Y and that particular X𝓗, adjusting (i.e. controlling) for all the other X's

GENERAL GOALS IN ANY MULTIVARIATE ANALYSIS

1. Best Fit
2. Most Parsimonious (Occams Razor)
   -use as simple model as possible
3. Biologically reasonable

CENTRAL QUESTION IN ANY MULTIVARIATE ANALYSIS

Does the model that includes the variable(s) in question tell us more about the outcome variable than does a model that does not include those variable(s)?

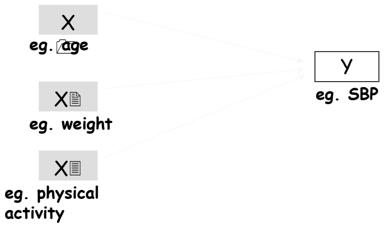
Y ~ X🗁, X🗎, X🗏 X🗐, X🗏

↑

Does addition of these 2 add significantly to prediction of outcome variable?

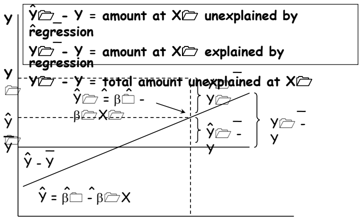
MULTIPLE REGRESSION

Relationship



Y , X🗁 , X🗎 , X🗏 are continuous variables

ANOVA Table for Linear Regression



ŷ🗁_- Y = amount at X🗁 unexplained by regression

Y🗁 - Ȳ = amount at X🗁 explained by regression

Y🗁 - Ȳ = total amount unexplained at X🗁

For one observation, it is clear that :

Y🗁 - Ȳ = (Ŷ🗁 - Ŷ🗁) + (Y🗁 - Y)

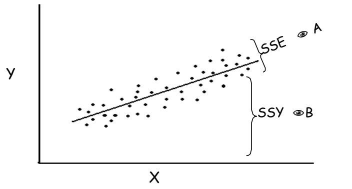
It can be shown that:

$$\sum_{♓=1}^{■} (y🗁 - ☒)^2 \quad \sum_{♓=1}^{■} (\hat{y}_1 - y_1)^2 \quad \sum_{♓=1}^{■} \hat{\ } (y_1 - ☒)^2$$

SSY = SSE + SSR

SSY	SSE	SSR
II	II	II
Total Sum of Squares about the mean	Sum of Squares due to error	Sum of Squares explained by Regression
II	II	
Total Unexplained variation	Residual Sum of Squares	
	II	
	Sum of Squares about the Regression Line	

Visual Interpretation of SSY and SSE



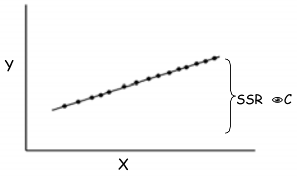
SSE represents the variation of the data points seen from A

SSY represents the variation of the data points seen from B

When β🗁 = 0, then SSE = SSY

The steeper the regression line, the greater will be SSY compared to SSE

Visual Interpretation of SSR



SSR may be interpreted as the variation of hypothetical data points sitting exactly on the regression line, when seen from C

General Approach for Assessing the Significance of Predictor Variable(s) in the Model:

i) Compare Observed vs Expected (with)

= SSE (with)

ii) Compare Observed vs Expected (without)

= SSE (without)

iii) ✂ = SSE (without) - SSE (with)

Application of the General Approach to Multiple Regression Model

i) **Compare O vs E(with):**

$$\sum_{♓=1}^{■} \{Y🗁 \hat{-} Y🗁(\text{with})\}🗎 = SSE(\text{with})$$

Observed value of response variable y

Predicted(or expected) value of y, according to the model which includes the predictor variable(s) in question

ii) **Compare O vs E(without):**

$$\sum_{♓=1}^{■} \{Y🗁 \hat{-} Y🗁(\text{without})\}🗎 = SSE(\text{without})$$

Observed value of response variable y

Predicted(or expected) value of y, according to the model which excludes the predictor variable(s) in question

iii) **Difference between i) and ii):**

✍ = SSE(without) - SSE(with)

$\boxed{\gamma🗎 = SSR/SSY}$

So r🗎 is the proportion of the total variation which can be explained by the linear regression model.

e.g. for r = 0.5, only 25% of the total observed variation can be explained by the linear regression model. It takes r > 0.7 to make r🗎 > 0.5, i.e. more than 50% of the total variation explained by the linear regression model.

When r = 0, none of the observed variation can be explained by the linear regression model.

When r = 1, all of the observed variation can be explained by the linear regression model.

ANOVA Table For Straight-Line Regression

Source	Sum of Squares (SS)	Degrees of Freedom	Mean square (MS)	Variance Ratio
Regression	SSR	1	MSR = (SSR/1)	F = MSR/MSE
Residual	SSE	n – 2	MSE = (SSE/n-2)	
Total	SSY	n - 1		

$\hat{\sigma}$🗎 = MSE

γ🗎 = SSR/SSY

BASIC PREMISE

Outcome : $\boxed{X🗁\ X🗎}$ … $\boxed{X_{p+1}\ ,}$ X_{p+2}

[Y] $\underbrace{\quad}_{p}$

Reduced Model

Full Model

For Multiple Regression : Use F Statistic

For Logistic Regression : Use LR Statistic (likelihood ratio)

STATISTICAL ASSUMPTIONS

Assumptions:

i) E (Y| x🗁, x🗎, … x&) ≡ $\mu_{Y|\ x🗁, x🗎, \ldots\ x\&}$

= β🗀 + β🗁x🗁 + … + β&x& **{Assumption of Linearity}**

ii) VAR (Y|x🗁 , x🗎 , … x&) ≡ $\sigma🗎_{Y|\ x🗁, x🗎, \ldots\ x\&}$ = σ🗎 **{Assumption of Homoscedasticity}**

iii) 🖎 distribution

TESTS available

i) Test for Significant Overall Regression

Y : X🗁 X🗎 X🗎

ii) Test for Addition of Single Variable

- Partial F Test

$Y : \underbrace{X_1\ , X_2}$ $\underbrace{X_3}$

p=2; predictor variable already in the model   k=1; additional variable in question

iii) Test for additional of a group of variables

- Multiple-Partial F test

$Y : \underbrace{X_1}$ $\underbrace{X_2\ , X_3}$

p=1; predictor variable already in the model   k=2; additional variables in question

Important Applications of:

A) The Partial F-test

Y : $\boxed{C🗁\ C🗎\ ..}$ C_p $\boxed{S}$

p controlling variables (confounders)   main study variables

b) The Multiple-Partial F-test

$Y : \underbrace{X_1\ ,\ X_2\ ,}\ X_3$ $\underbrace{X_1^2\ ,\ X_2^2,}\ X_3^3$

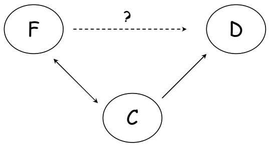
variables already in the model   additional "higher order" variables in question

$Y : \underbrace{X_1\ ,\ X_2\ ,}\ X_3$ $\underbrace{X_1X_2\ ,\ X_1X_3\ ,\ X_2}X_3$

variables already in the model   additional interaction variables in question

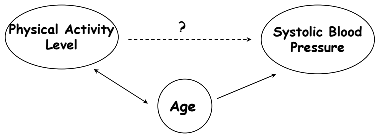
CONFOUNDING AND INTERACTION

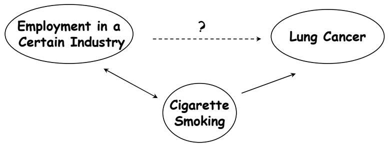
Confounding



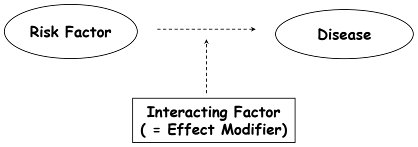
Confounding is the distortion of a risk factor-disease relationship brought about by the association of other factors with both risk factor and disease, the latter associations with the disease being causal. These factors are called Confounding Factors or "Confounders"

Examples of Confounding



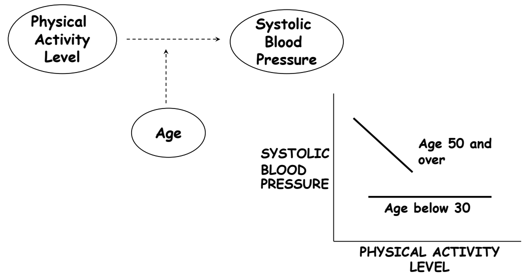


INTERACTION

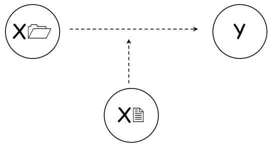


Interaction exists when the primary relationship of interest between a risk factor and a disease is different at different levels of the interacting factor (also known as effect modifier)

Examples of Interaction



How to Detect Existence of Confounding & Interaction:



Y = β🗀 + β🗁X🗁 + β🗎X🗎 + β🗏(X🗁X🗎)

Product term

Statistical testing with the F tests could be used to evaluate the existence of interaction for any given model. In the above example, a partial F test of H🗀 : β🗏 = 0 could be used.

CONFOUNDING

Have 2 models

Y = β🗀 + β🗁x🗁 + ε

Y = β*🗀 + β*🗁 x🗁 + β*🗎 x🗎 + ε

with respect to X🗎,

β🗁 = β crude (ignore X🗎)

β*🗁 = β adjusted (adjust for X🗎)

β crude ≠ β adjusted if confounding is present

CONFOUNDING AND INTERACTION

- **Confounding and interaction are different phenomena**
- ***A variable may be both a confounder and an interactor, or only one of the two or neither***
- **Interaction should be assessed before confounding**
- **The use of adjusted estimate that controls for confounding is recommended only when there is no meaningful interaction**
- **If strong interaction is found, an adjustment for confounding is inappropriate. Instead there should be separate results for separate categories of the effect modifier.**

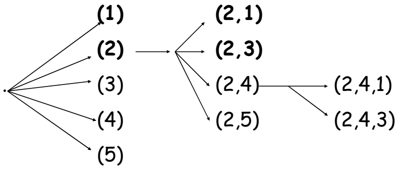
AUTOMATIC ELIMINATION STRATEGIES

With a large number of inter-related predictor variables, it often becomes quite difficult to sort out the meaning of the individual regression coefficients

- Need for Automatic Elimination Strategies

AUTOMATIC ELIMINATION STRATEGIES

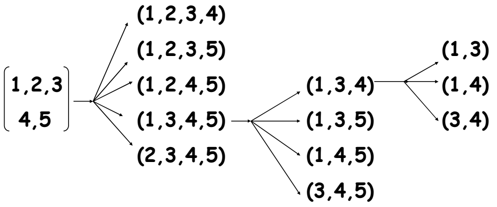
a) Step-up (=Forward selection) Strategy



The equation starts out empty and IVs are added one at a time provided they meet the statistical criteria for entry

AUTOMATIC ELIMINATION STRATEGIES

b) Step-down (= Backward Elimination) Strategy



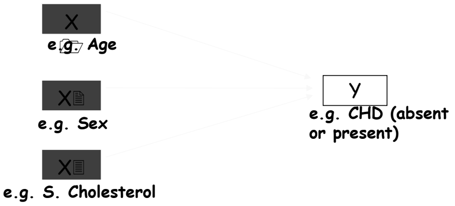
The equation starts out with all IVs entered and they are deleted one at a time if they do not contribute significantly to regression

AUTOMATIC ELIMINATION STRATEGIES

c) Stepwise Regression

- a compromise between a) and b)
- equation starts out empty and IVs are added one at a time if they meet statistical criteria but they may also be deleted at any step where they no longer contribute significantly to regression.
- considered the surest path to the best prediction equation

Logistic Regression



Y = Binary Dependent Random variable; Y = 0 or 1

X's = Independent (fixed) variables;
Categorical (e.g. sex) or
continuous (e.g. age, S. Cholesterol)

Mathematical Model for Logistic Regression

a) Explicit form:

$$\Pr\{ Y = 1 | x\} = \frac{e^{\beta\text{🗀} + \beta\text{🗁}x}}{1 + e^{\beta\text{🗀} + \beta\text{🗁}x}} = \frac{1}{1 + e^{-(\beta\text{🗀} + \beta\text{🗁}x)}}$$

b) Logit Form

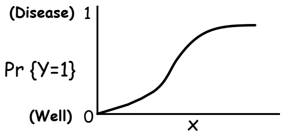
$$\text{Logit} \Pr\{ Y = 1|x \} = \beta_0 + \beta_1 x$$

$$\equiv \quad Y = \beta_0 + \beta_1 x_1 + \dots$$

Remember! (Y = a + bx)

LOGISTIC REGRESSION

Relationship between Y and X



This S shaped curve is that of a logistic distribution
- thought to agree with real world situation

Variation of the logistic relationship:

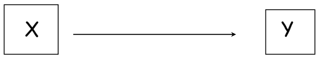
One Independent Dichotomous Variable

Exporure (0,1) **Outcome (0,1)**

[Independent Variable]	**[Dependent variable]**
0 = Lower Risk → Nonsmoker	**0 = Normal → well**
1 = Higher Risk → Smoker	**1 = Not normal → sick**

Variations of the Logistic Relationship:

One Independent Polytomous Variable



Smoking Status
1 = Nonsmoker
2 = Light Smoker
3 = Heavy smoker

Disease/ Non disease

Have to convert to <u>DUMMY VARIABLES</u>

CONSTRUCTION OF DUMMY VARIABLES

	<u>Dummy Variables</u>	
Smoking Status	Smoke (1)	Smoke (2)
1 = Non smoker	0	0
2 = Light smoker	1	0
3 = Heavy smoker	0	1

MULTIPLE LOGISTIC REGRESSION MODEL

a) Explicit Form:

$$\Pr\{Y=1|x_1, x_2, x_3, \dots x_p\} = \frac{1}{1 + e^{-(\beta\text{🗀} + \beta\text{🗁} X\text{🗁} + \beta\text{🗎}X\text{🗎} + \beta\text{🗏}X\text{🗏} + \dots \beta\text{■} x\text{■})}}$$

B) Logit Form:

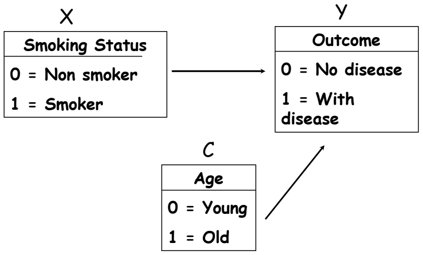
$$\textbf{Logit} = \beta\text{🗀} + \beta\text{🗁}\, x\text{🗁} + \beta\text{🗎}\, x\text{🗎} + \beta\text{🗏}\, x\text{🗏} + \dots + \beta_p\, x_p$$

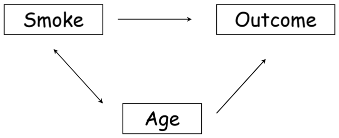
where Logit P = Log_e (p / 1- p)

compare with

$$Y = \beta\text{🗀} + \beta\text{🗁}\, x\text{🗁} + \beta\text{🗎}\, x\text{🗎} + \beta\text{🗏}\, x\text{🗏} + \dots + \beta_p\, x_p$$

MULTIPLE LOGISTIC REGRESSION





Is Age an Independent Risk Factor?

Is Age a Confounder?

Is Age an Interactor / Effect Modifier?

Is Age an Interactor / Effect Modifier?

Use Product-Term i.e. Age Smoke

e.g.

$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ … Reduced model

$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$ … Full model

Compare Full Model with Reduced Model

COMPUTER SELECTION OF PREDICTORS

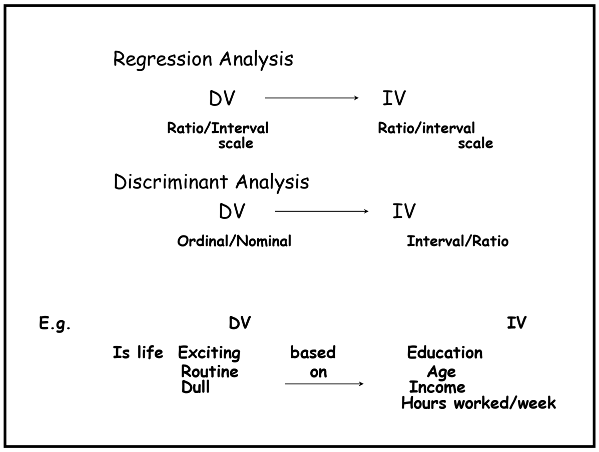
- Forward LR
- Backward LR
- Stepwise

OTHER MULTIVARIATE ANALYSIS

- **Discriminant Function Analysis**
- **Factor Analysis including Principal Component Technique**
- **Cluster Analysis**

DISCRIMINANT FUNCTION ANALYSIS

- A technique for deciding into which category of a variable a case is most likely to fall i.e. to predict group membership
- Compute “ discriminant scores” for each case to predict what group it is in
- Normally, have only two discriminant groups



Regression Analysis

DV → IV

DV	IV
Ratio/Interval scale	Ratio/interval scale

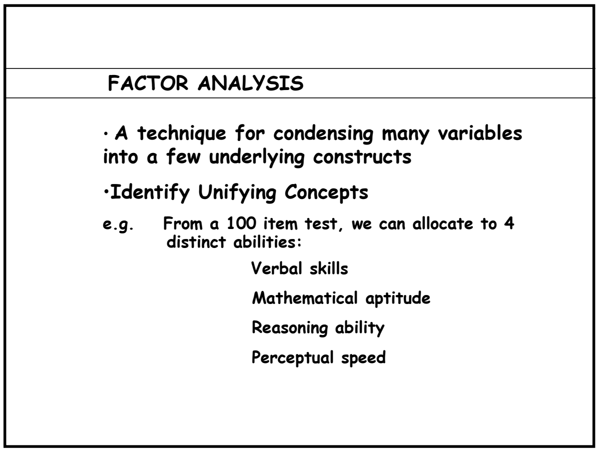
Discriminant Analysis

DV → IV

DV	IV
Ordinal/Nominal	Interval/Ratio

E.g.

	DV		IV
Is life	Exciting Routine Dull	based on →	Education Age Income Hours worked/week



FACTOR ANALYSIS

- **A technique for condensing many variables into a few underlying constructs**
- **Identify Unifying Concepts**

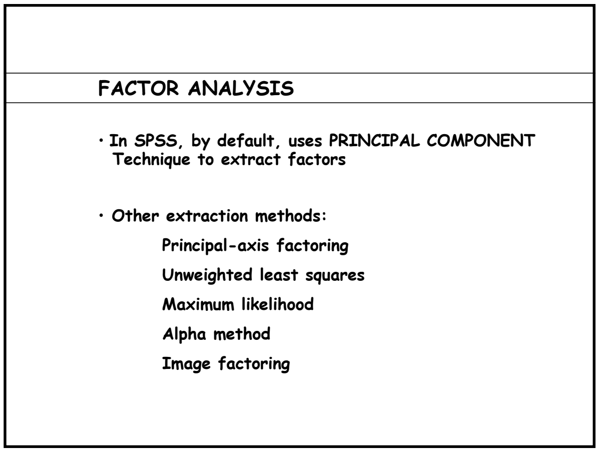
e.g. From a 100 item test, we can allocate to 4 distinct abilities:

Verbal skills

Mathematical aptitude

Reasoning ability

Perceptual speed



FACTOR ANALYSIS

- **In SPSS, by default, uses PRINCIPAL COMPONENT Technique to extract factors**
- **Other extraction methods:**

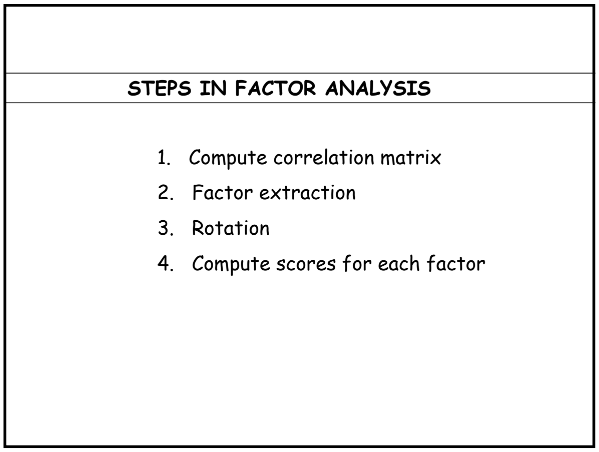
Principal-axis factoring

Unweighted least squares

Maximum likelihood

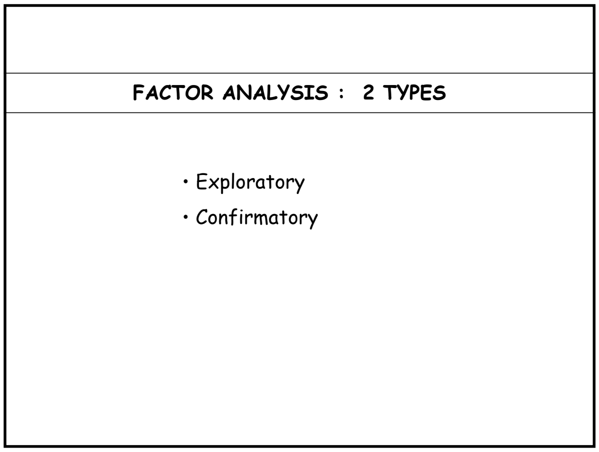
Alpha method

Image factoring



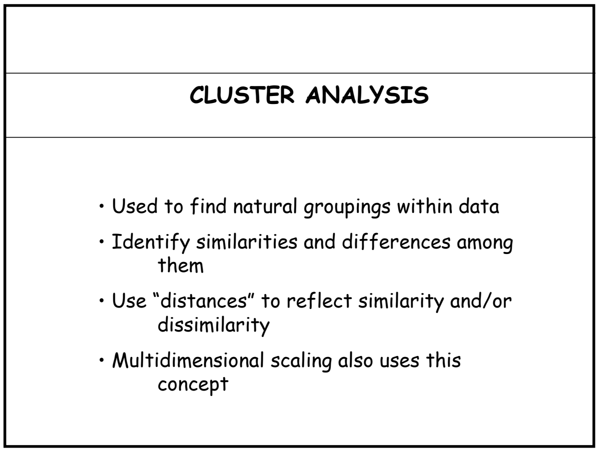
STEPS IN FACTOR ANALYSIS

1. Compute correlation matrix
2. Factor extraction
3. Rotation
4. Compute scores for each factor



FACTOR ANALYSIS : 2 TYPES

- Exploratory
- Confirmatory



CLUSTER ANALYSIS

- Used to find natural groupings within data
- Identify similarities and differences among them
- Use “distances” to reflect similarity and/or dissimilarity
- Multidimensional scaling also uses this concept

LOGLINEAR REGRESSION

- Extension of CrossTabulation and Chi Square Statistic for Independence
- Difficulty in crosstabulating for > 2 variables
- Use a LOGLINEAR Model

OTHER SPSS STATISTICS FUNCTIONS

- General Linear Model
- Reliability Analysis
- Multidimensional Scaling
- Probit Analysis
- Survival Analysis
- ETC

GENERAL LINEAR MODEL

- GLM Factorial Analysis
- GLM Univariate Analysis - combination of Regression and ANOVA
- GLM Multivariate Analysis
- GLM Repeated Measures
- Variance Components

THANK YOU